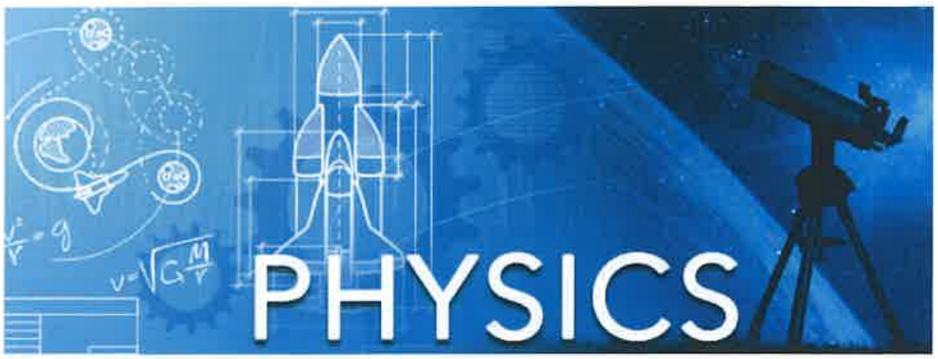
AP PHYSICS 1



SUMMER ASSIGNMENT READING & SAMPLE PROBLEMS

VECTORS AND SCALARS

Physics deals with many quantities that need to be measured. All the quantities that we will be studying throughout the year fall into one of two categories: scalars and vectors. Scalar quantities are those which can be expressed in terms of a magnitude (number or size) with the appropriate units. Time, mass, distance and speed are examples of scalar quantities.

EXAMPLES:

Time: 10 seconds, 2.5 minutes, 4.2 hours

Mass: 15 grams, 30 Kg

Distance: 12 inches, 2.54 cm, 25 miles

Speed: 50 mi/hr (miles per hour), 8 m/s (meters per second), 125 Km/hr (kilometers per hour)

Vector quantities are those which can be expressed in terms of a magnitude (with units) and **direction.** Displacement, velocity and force are the vector quantities that will be discussed in this unit. (With each unit we study, we will be introduced to more scalar and vector quantities.

EXAMPLES:

Displacement: 14 meters south, 37.4 Km northeast

Velocity: 60 m/s northwest, 150 Km/hr north

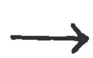
Force: 220 Newtons east, 47 Newtons west

NOTE: A force is defined as a push or pull on an object. A newton (named after physicist and mathematician Isaac Newton) is the metric unit of force. One newton is equal to 0.2248 pounds (lbs). We will deal almost exclusively with metric units in AP Physics. A newton is abbreviated with a capital **"N"**; do not confuse this "N" with the direction north, which is also commonly abbreviated with an "N". They should not be easily confused; one is a unit, one is a direction.

Since direction is the defining characteristic of all vector quantities and in drawing vector diagrams, vectors are represented by an arrow, which is drawn to scale in a particular direction. The larger the vector quantity, the larger the arrow.

EXAMPLES:

Scale: 1.0 cm = 10 meters

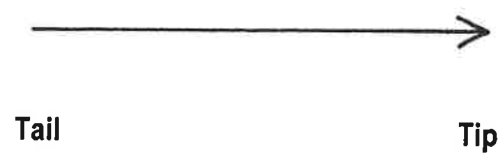


10 meters east

20 meters north

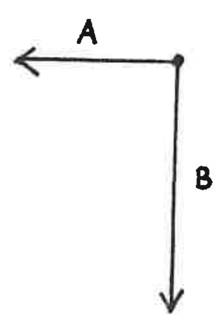
30 meters west

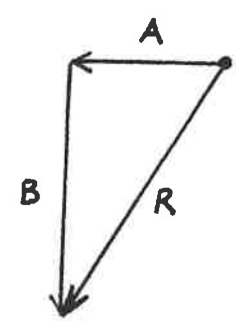
Most problems in physics will contain more than one vector. It is then useful to determine the net result of all these vectors. A **resultant** is a single vector which equals the effect of all these individual vectors combined. Because vectors have directions, one cannot simply add the magnitudes of all these individual vectors together to find the resultant. The "**Tip to tail rule**" explains how to determine the direction of the resultant of two or more vectors. The "tip to tail rule" is as follows: to determine the direction of the resultant, arrange all vectors tip to tail **(without changing direction)** so that the tip of one vector touches the tail of another. The order in which the vectors are rearranged does not matter. In rearranging vectors, it is alright if the vectors overlap or cross over one another. After the vectors are arranged tip to tail, there should be only two "loose ends", one tail and one tip. (If there are more than two loose ends, then you need to continue rearranging the vectors until there are only two loose ends.) The resultant is then drawn from the **free tail to the free tip**. The resultant vector is commonly labeled **"R"**. The magnitude (size) of the resultant depends on the orientation of the original vectors and will be discussed shortly.



EXAMPLE 1: Using the "tip to tail rule", rearrange the following combinations of vectors and draw in (and label) the resultant, R.

1)





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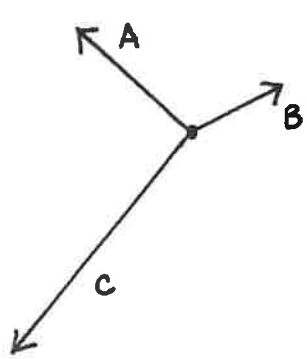
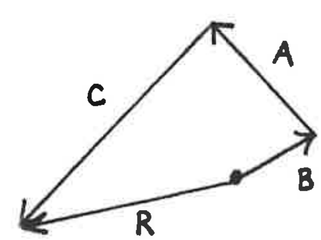
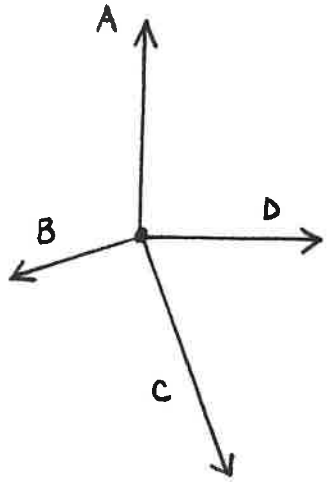
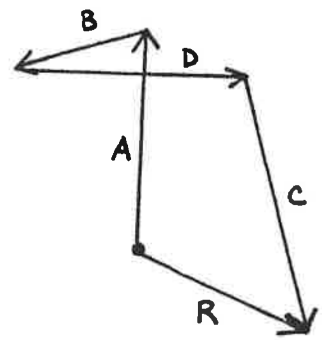


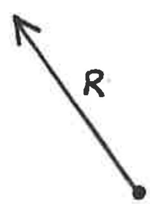


3) 

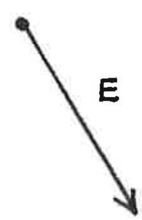


Forces are examples of vector quantities which will be used in many topics in AP Physics. There are two important terms which apply to problems involving forces. **Equilibrium** is a state of balance in which the net (resultant) force acting on an object equals zero. An **equilibrant** is a force which is **equal in magnitude but opposite in direction** to the resultant force. The equilibrant force is commonly labeled **"E".** An equilibrant force acting with a resultant force produces equilibrium. For example, if the resultant of several vectors has a magnitude and direction of 36 Newtons due west, the equilibrant force has a magnitude and direction of 36 Newtons due east. The numerical part of the answer is the same, just the direction is opposite.

EXAMPLE 2:

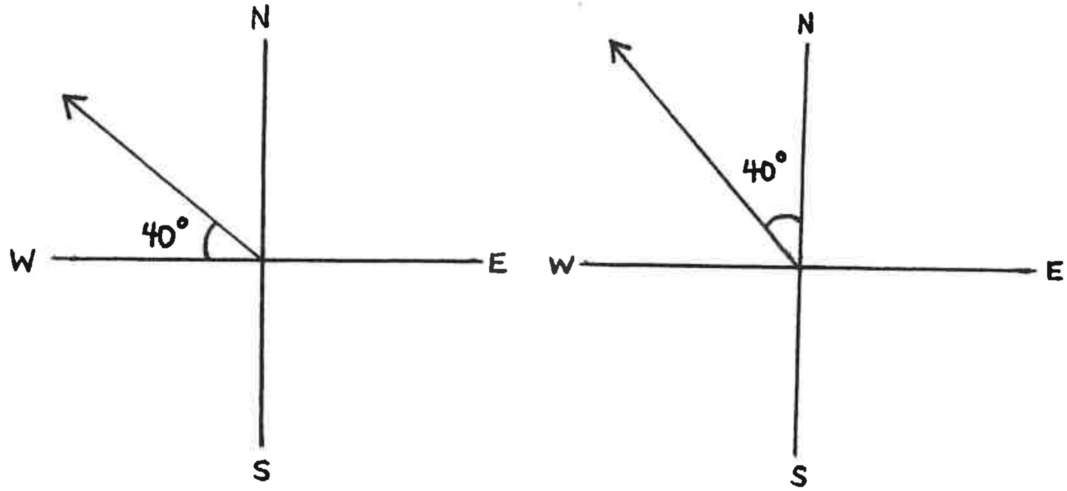


Resultant force
(20 Newtons northwest)



Equilibrant force
(20 Newtons southeast)

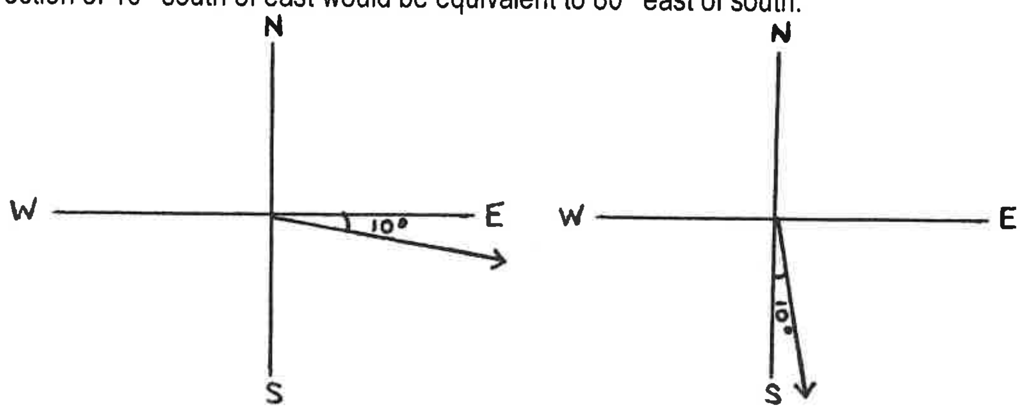
Since vectors are quantities that contain a direction, it is important to be familiar with the different ways in which directions can be expressed. In some cases, a general geographic direction such as "northwest" or "southeast" is adequate to describe the direction of a vector. More often, students will be expected to express their answers with very specific directions. The two most common ways to express direction involve geographic descriptions (with an appropriate angle) or as angles measured counterclockwise from the positive x axis. In dealing with geographic directions, one must realize that 40° north of west is not equal to 40° west of north. While both vectors would fall in the same general northwest quadrant, they are entirely different directions. If a vector is directed 40° north of west, that means that the vector is tilted 40° northward, measured relative to the west (-x) axis. A vector directed 40° west of north, means that the vector is tilted 40° west, measured relative to the north (+y) axis.



40° north of west

40° west of north.

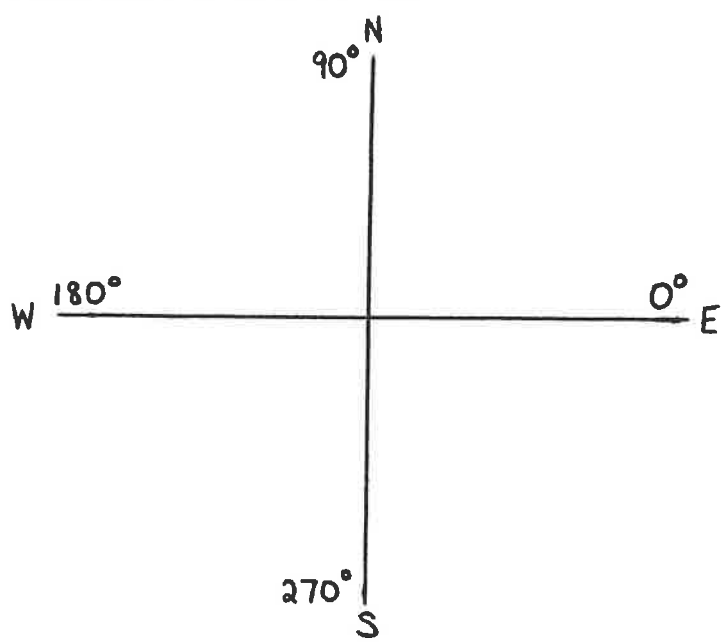
Every direction has several ways in which to be expressed. A vector directed at 40° north of west would be equivalent to a direction of 50° west of north. These two geographic descriptions represent exactly the same direction. (Note that the angles are complementary angles!) A direction of 10° south of east would be equivalent to 80° east of south.



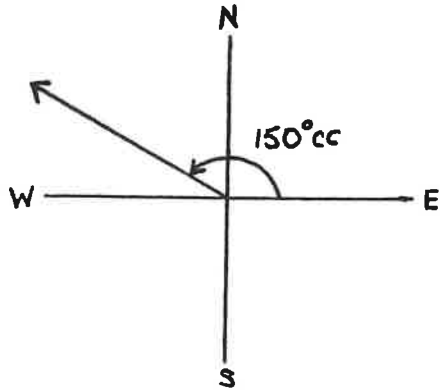
10° south of east

10° east of south

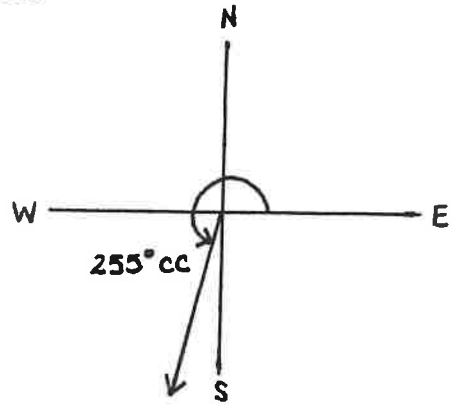
One way in which to avoid any geographic wording altogether is to express directions simply as an angle measured counterclockwise from the positive x axis. This way, all directions will be expressed as angles between 0° and 360°. Zero degrees (or 360°) would be equal to the direction east, 90° would be equivalent to the direction north, 180° would be equivalent to the direction west and 270° would be equivalent to the direction south.



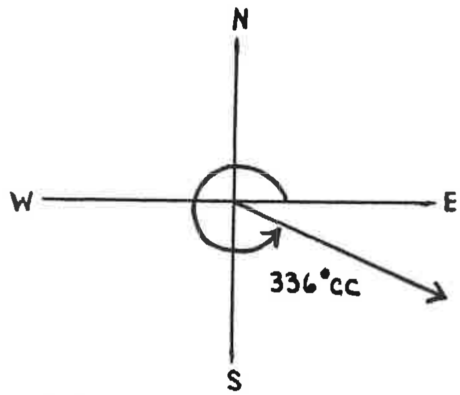
Angles between 0° and 90° would be in the first (northeast) quadrant, from 90° to 180° would be in the second (northwest) quadrant, from 180° to 270° would be in the third (southwest) quadrant, and 270° to 360° would be in the fourth (southeast) quadrant. So, for example, a geographic direction of 30° north of west would be equivalent to a counterclockwise (symbolized cc) angle of 180° - 30° = 150° cc. A geographic direction of 15° west of south would be equivalent to a counterclockwise angle of 270° - 15° = 255°. A geographic direction of 24° south of east would be equivalent to a counterclockwise angle of 360° - 24° = 336° cc.



30° north of west = 150° cc



15° west of south = 255° cc°

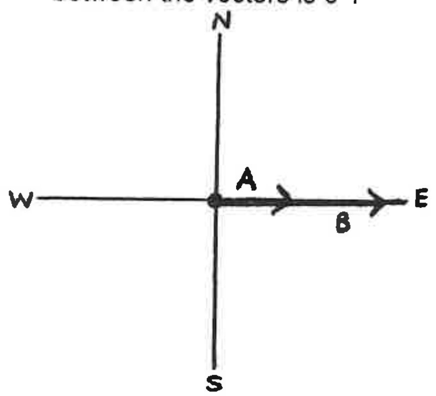


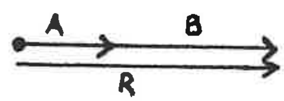
24° south of east = 336° cc

VECTOR ALGEBRA

The magnitude (size) of a resultant depends on the orientation of the original vectors. Vector diagrams are very useful to visualize the orientation of the vectors in each problem. It is strongly suggested to start each vector diagram with a directional (coordinate) axis, so each vector can be drawn in the appropriate geographic directions. To mathematically determine the magnitudes of resultant vectors, use the following strategies:

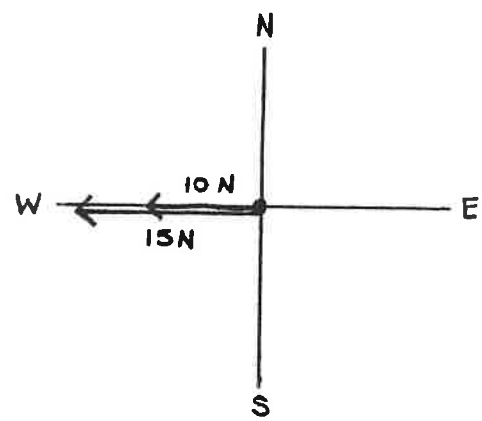
A. **For vectors acting in the same direction, add the magnitudes of the individual vectors to Find the magnitude of the resultant.** When vectors are in the same direction, the angle between them is defined as **zero degrees (0°).** The **maximum** resultant occurs when the angle between the vectors is 0°.

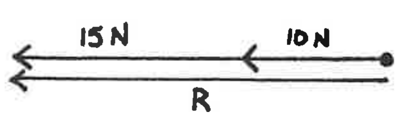




Vectors arranged tip to tail

EXAMPLE 1:
Two concurrent forces (act on the same object at the same time) of 10 Newtons west and 15 Newtons west pull on an object. Draw a labeled vector diagram and calculate the magnitude and direction of the resultant force.

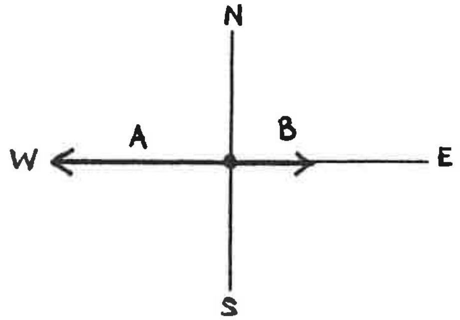


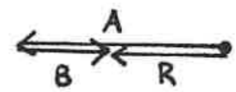


$R = A + B$

$R = 10N + 15N = 25N$ West

B. For vectors acting in opposite directions, subtract the magnitudes (large number minus small number) of the individual vectors to find the magnitude of the resultant. When vectors are in opposite directions, the angle between them is defined as **180 degrees (180°).** The **minimum** resultant occurs when the angle between the vectors is 180°.



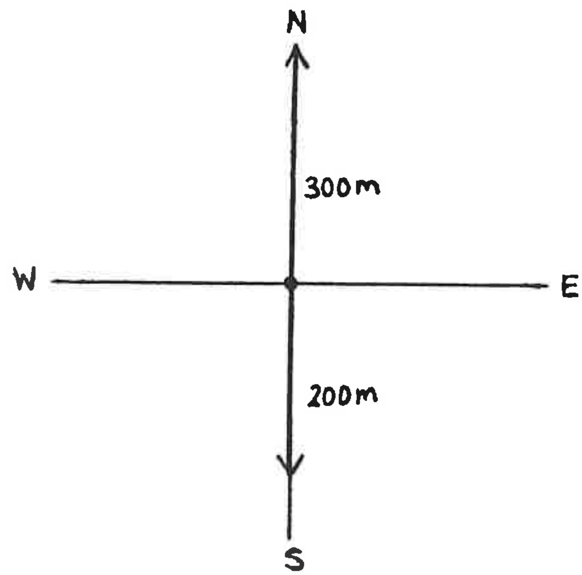


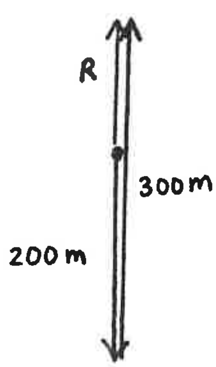
Vectors arranged tip to tail

The angles of 0° and 180° are very important because since they represent the angles at which the largest (maximum) and smallest (minimum) resultants occur, all other angles must yield resultants which fall between the maximum and minimum values.

EXAMPLE 2:

A person walks 200 meters south, then turns around and runs 300 meters north. Draw a labeled vector diagram and calculate the magnitude and direction of the person's resultant displacement.



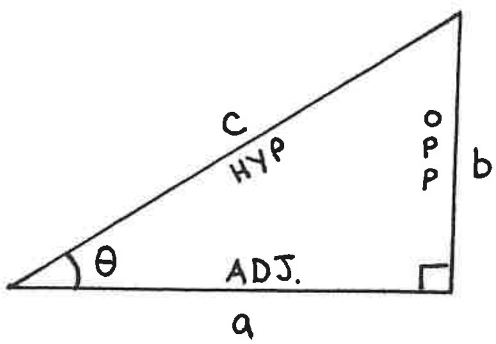


$R = A - B$

$R = 300\text{m} - 200\text{m} = 100\text{m}$ North

C. For vectors arranged at right angles (90°), use the PYTHAGOREAN THEOREM and SOHCAHTOA to find the magnitude and direction of the resultant.

$$a^2 + b^2 = c^2$$

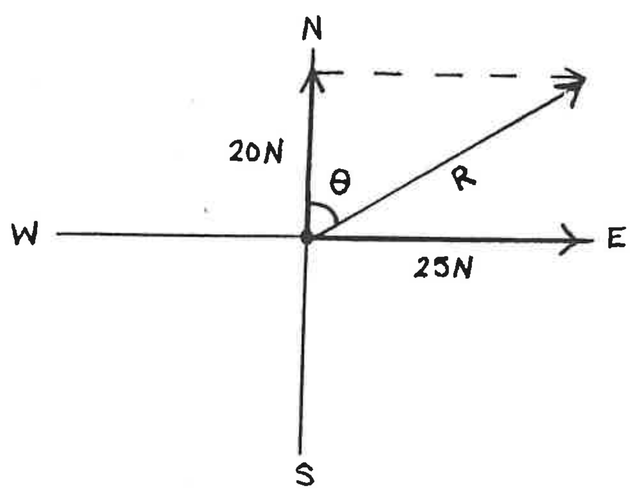


$\sin\theta = \dfrac{\text{Opposite}}{\text{Hypotenuse}}$

$\cos\theta = \dfrac{\text{Adjacent}}{\text{Hypotenuse}}$

$\tan\theta = \dfrac{\text{Opposite}}{\text{Adjacent}}$

EXAMPLE 3:
Two forces of 20 Newtons north and 25 Newtons east act on an object. Draw a labeled vector diagram and calculate the magnitude and direction (including a specific angle, θ) of the resultant force and the equilibrant force.



$a^2 + b^2 = c^2$

$(20N)^2 + (25N)^2 = c^2$

$400N^2 + 625N^2 = c^2$

$c^2 = 1025N^2$

$c = 32.0N$

* Equilibrant = 32.0N,
39° S of W or 219° cc

$\tan\theta = \dfrac{opp}{adj} = \dfrac{25N}{20N}$

$\tan\theta = 1.25$

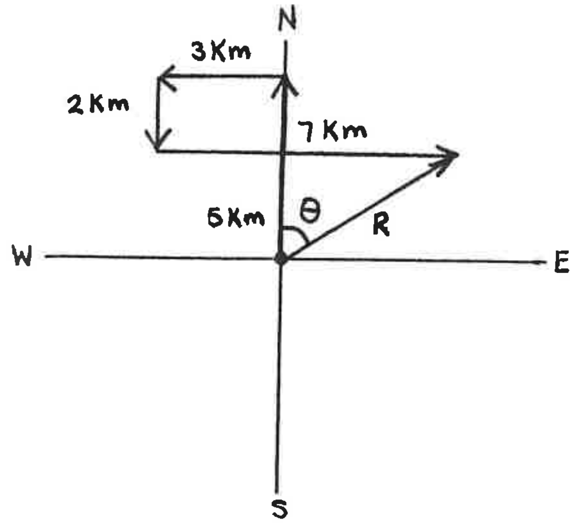
$\theta = \tan^{-1}(1.25)$

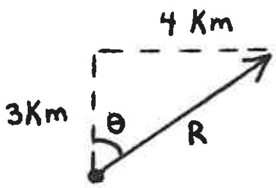
$\theta = 51°$ east of north

(or 39° north of east,
39° cc)

EXAMPLE 4:

A student drives 5 Km north, 3 Km west, 2 Km south, then 7 Km east. Draw a labeled vector diagram and calculate the magnitude and direction (including a specific angle, θ) of the student's resultant displacement.





$a^2 + b^2 = c^2$

$(3\,Km)^2 + (4\,Km)^2 = c^2$

$c = 5\,Km$

$\tan\theta = \frac{opp}{adj}$

$\tan\theta = \frac{4\,Km}{3\,Km} = 1.333$

$\theta = 53^\circ$ east of north

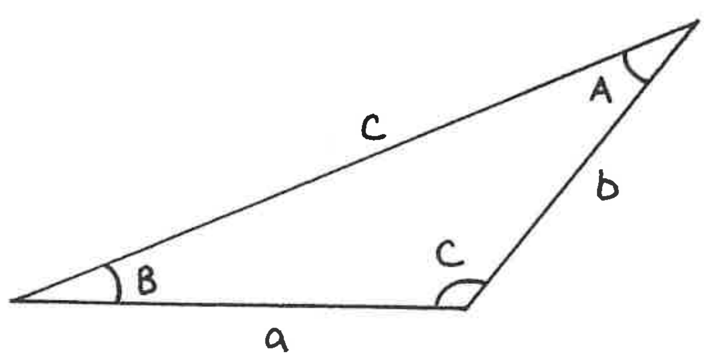
(or 37° north of east, 37°cc)

D. For vectors arranged at angles other than 0°, 180 or 90°, use the laws of sines and cosines to find the magnitude of the resultant.

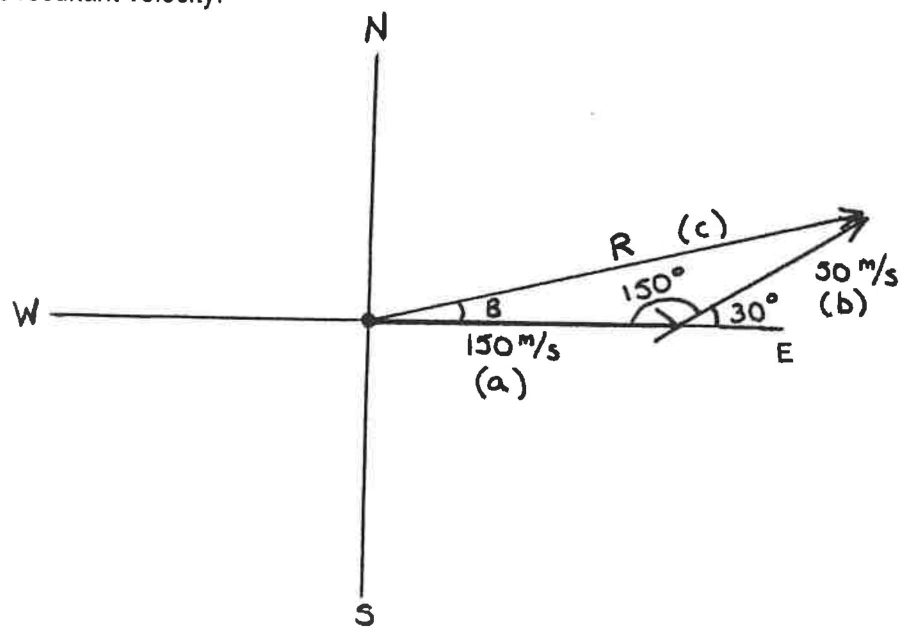
Law of sines: $c^2 = a^2 + b^2 - (2ab\cos C)$

Law of cosines: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

NOTE: When using the laws of sines and cosines, after drawing your vector diagram, it is easiest to assign the letters "a" and "b" for the two original individual vectors and "c" for the resultant vector.



EXAMPLE 5:
A plane flies at a velocity of 150 m/s east while the wind blows at 50 m/s north of east at 30°. Draw a labeled vector diagram and calculate the magnitude and direction (including a specific angle, θ) of the plane's resultant velocity.



$c^2 = a^2 + b^2 - (2ab\cos c)$

180° − 30°

$c^2 = (150\frac{m}{s})^2 + (50\frac{m}{s})^2 - (2 \cdot 150\frac{m}{s} \cdot 50\frac{m}{s} \cos 150°)$

$c^2 = 22500\frac{m^2}{s^2} + 2500\frac{m^2}{s^2} - (15000\frac{m^2}{s^2} \cdot (-0.866))$

$c^2 = 25000\frac{m^2}{s^2} - (-12990.4\frac{m^2}{s^2}) = 37990.4\frac{m^2}{s^2}$

$c = 194.9\frac{m}{s}$

$$\frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{50\frac{m}{s}}{\sin B} = \frac{194.9\frac{m}{s}}{\sin 150°}$$

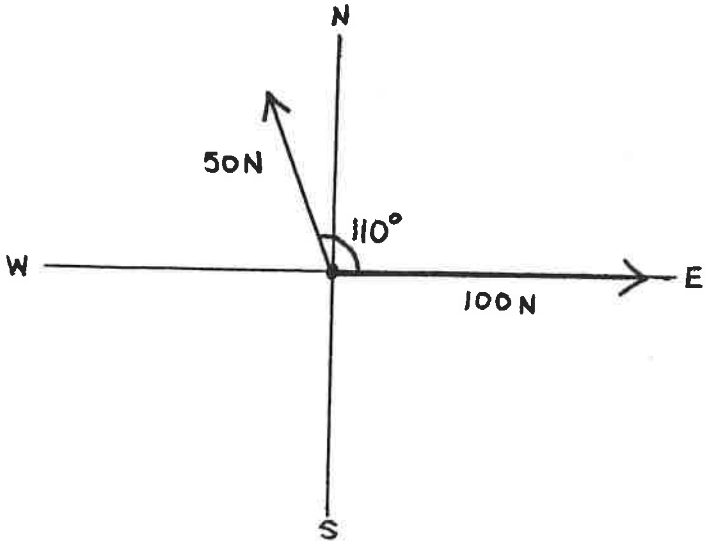
$\sin B = 0.1283$

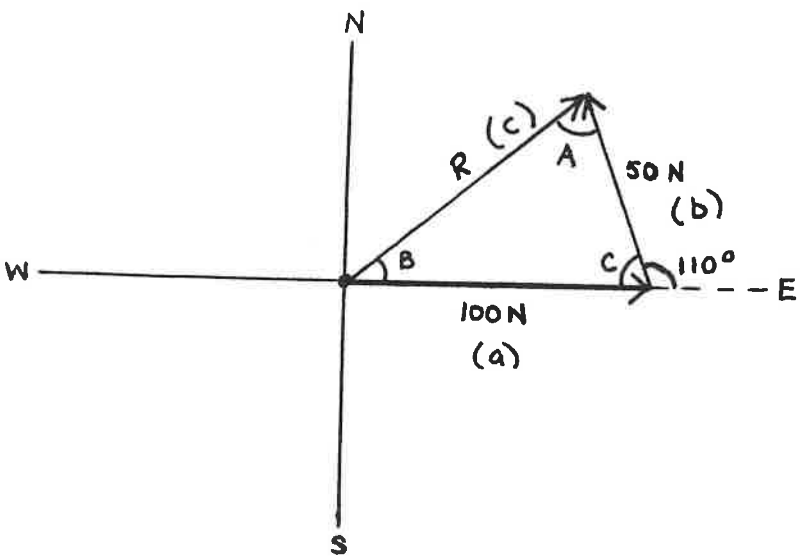
$B = 7°$

θ = 7° north of east
(or 83° east of north, 7° cc)

EXAMPLE 6:

Two forces of 100 N and 50 N act on a body, as shown below. Calculate the magnitude and direction of the resultant force when the angle between the forces is 110°.





$$\frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{50N}{\sin B} = \frac{95.3N}{\sin 70°}$$

$\sin B = 0.4930$

$B = 30°$

$\theta = 30°$ north of east

(or 60° E of N, 30° cc)

$c^2 = a^2 + b^2 - (2ab\cos c)$

$180° - 110°$

$c^2 = (100N)^2 + (50N)^2 - (2 \cdot 100N \cdot 50N \cos 70°)$

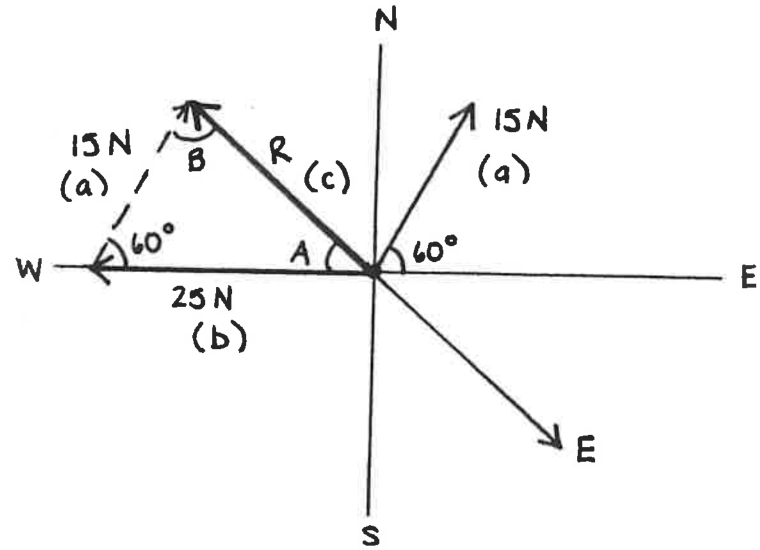
$c^2 = 10000N^2 + 2500N^2 - (10000N^2\ (0.342))$

$c^2 = 12500N^2 - (3420N^2) = 9080N^2$

$c = 95.3N$

EXAMPLE 7:

A force F_1 = 25 N due west and another force F_2 = 15 N directed 60° north of east act on an object. Draw a labeled vector diagram and calculate the magnitude and direction of the resultant and equilibrant.



$$c^2 = a^2 + b^2 - (2ab\cos c)$$

$$c^2 = (15N)^2 + (25N)^2 - (2 \cdot 15N \cdot 25\,N \cos 60°)$$

$$c^2 = 225N^2 + 625N^2 - (750\,N^2\ (0.5))$$

$$c^2 = 850\,N^2 - 375N^2 = 475N^2$$

$$c = 21.8\,N$$

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$

$$\frac{15N}{\sin A} = \frac{21.8N}{\sin 60°}$$

$$\sin A = 0.5959$$

$A = 37°$ → $\theta = 37°$ north of west

(or 53° W of N, 143° cc)

R = 21.8N, 37° N of W

E = 21.8N, 37° S of E (323° cc)

OTHER VECTOR NOTATION

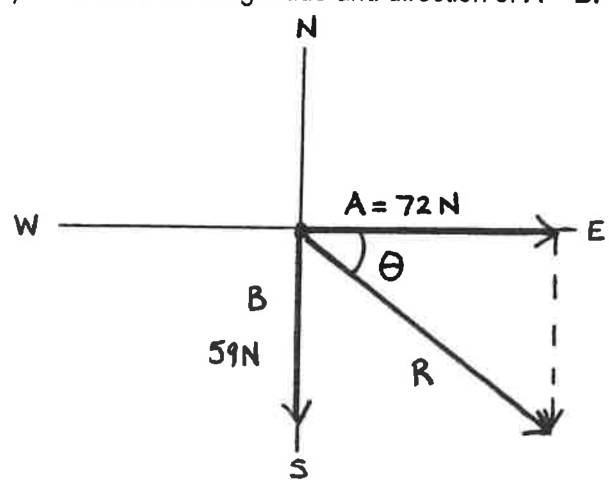
Many times vector quantities will just be assigned simple symbols such as **A, B, C** etc. These symbols will represent specific magnitudes and specific directions. If a vector symbol has a **negative sign** in front of it, that means that the magnitude of the vector has not changed but the direction is now opposite. For example, if vector **A** is a force vector with a magnitude of 68 Newtons directed 35° north of west (or 145° cc), then **–A** is equal to 68 Newtons directed 35° south of east (or 325° cc). When the resultant (R) of two vectors, **A** and **B**, is to be determined, is can be expressed as:

R = A + B

A + B (also known as **vector addition**) does not mean that you will necessarily be adding together the magnitudes of the vectors to find the resultant. Rather you will be performing **vector algebra**, meaning that depending on the orientation of the vectors, you may have to subtract, use the Pythagorean theorem of the laws of sines and cosines to find the magnitude of the resultant. **A + B** is not a literal mathematical representation telling you that addition is required; it just means you are finding the resultant of vectors **A** and **B**. The expression **A – B,** also known as **vector subtraction,** is therefore equal to **A + (-B).** You would find the resultant of vectors **A** and **–B**.

EXAMPLE 1:
Two forces, **A** and **B**, have the following magnitudes and directions:
A = 72 N due east
B = 59 N due south

(a) Determine the magnitude and direction of **A + B.**



$a^2 + b^2 = c^2$

$(72N)^2 + (59N)^2 = c^2$

$5184N^2 + 3481N^2 = c^2$

$c^2 = 8665N^2$

$c = 93.1N$

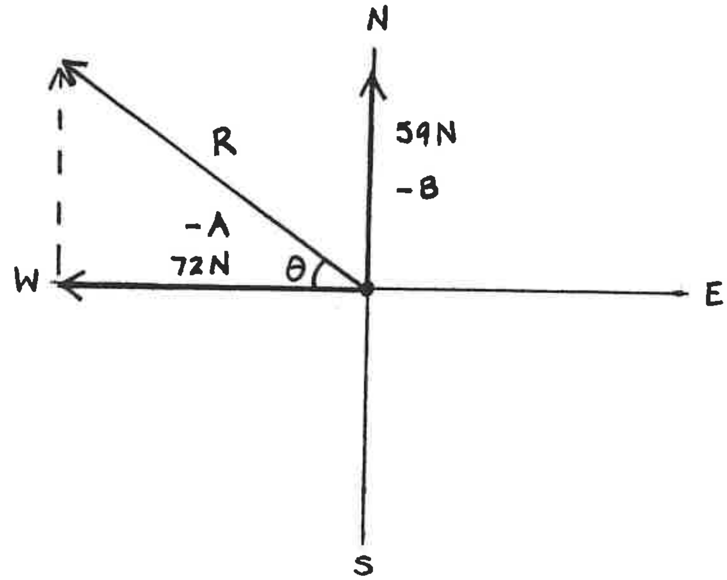
$\tan\theta = \frac{opp}{adj}$

$\tan\theta = \frac{59N}{72N} = 0.8194$

$\theta = 39°$ south of east

(or 51° east of south, 321° cc)

(b) Determine the magnitude and direction of **–A – B.**



$$a^2 + b^2 = c^2$$

$$(72N)^2 + (59N)^2 = c^2$$

$$c = 93.1N$$

$$\tan\theta = \frac{opp}{adj}$$

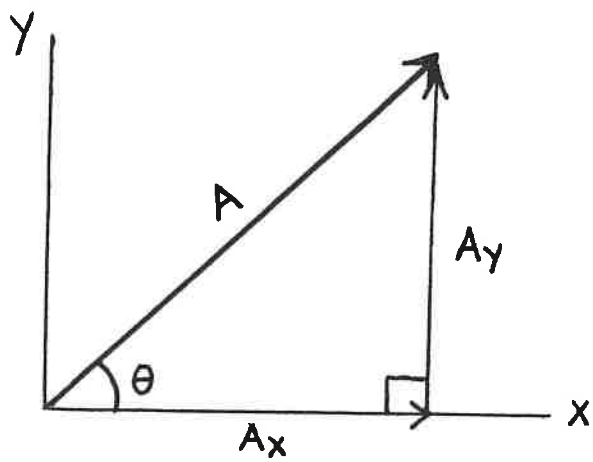
$$\tan\theta = \frac{59N}{72N}$$

$\theta = 39°$ north of west

(or 51° W of N, 141° cc)

VECTOR COMPONENTS

The first part of this unit dealt with finding the resultant from two or more individual vectors. The opposite process, in which one starts with a resultant vector and finds the individual vectors which make it up, can be useful as well. These separate vectors which make up the resultant vectors are known as **components.** Any resultant vector can be resolved into an unlimited number of components, but it is most beneficial to find the **perpendicular** components of the resultant. (The process of determining the components of a resultant vector is known as **vector resolution**.) If one visualizes the resultant as the hypotenuse of a right triangle, the two perpendicular components would be the two "legs" of the right triangle. If a resultant vector is denoted **A**, then the two components would be denoted $\mathbf{A_x}$ and $\mathbf{A_y}$. Since the x axis is the horizontal axis, $\mathbf{A_x}$ would be the component that is horizontal or parallel to the x axis. Since the y axis is the vertical axis, $\mathbf{A_y}$ would be the component that is vertical or parallel to the y axis. $\mathbf{A_x}$ and $\mathbf{A_y}$ are therefore perpendicular to each other. The two component vectors $\mathbf{A_x}$ and $\mathbf{A_y}$ will be drawn tip to tail with each other.



To find the magnitudes of $\mathbf{A_x}$ and $\mathbf{A_y}$, the following equations can be used:

$$\mathbf{A_x = A\cos\theta}$$

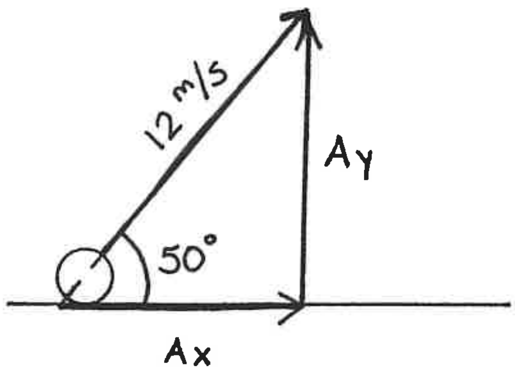
$$\mathbf{A_y = A\sin\theta}$$

where **A** is the magnitude of the resultant/hypotenuse and $\boldsymbol{\theta}$ is the angle between the resultant/hypotenuse and the **x axis.** These two equations are equivalent to SOHCAHTOA, they are just written differently. It is perfectly fine to use cos θ = adjacent/hypotenuse and sin θ = opposite/hypotenuse for components calculations, but you will find that the $\mathbf{A_x = A\cos\theta}$ and $\mathbf{A_y = A\sin\theta}$ notation will be used more often.

NOTE: If the angle, θ, is not measured relative to the x axis, then the equations $\mathbf{A_x = A\cos}$ and $\mathbf{A_y = A\sin\theta}$ will give you the wrong answers! SOHCAHTOA is always safe to use, because it doesn't matter from which axis the angle is measured. The "opposite" component will always use sine and the "adjacent" component will always use cosine.

Also, for this unit, we will be expressing all vector quantities with the general symbol, **A.** Starting with the next unit, different vector (and scalar) quantities will have different symbols assigned to them. For example **F** will represent force, **v** will represent velocity etc.

EXAMPLE 1:
A soccer ball is kicked at 12 m/s at a 50° angle relative to the ground. Calculate the horizontal and vertical components of the velocity.



$A_x = A\cos\theta$

$A_x = (12\frac{m}{s})\cos 50°$

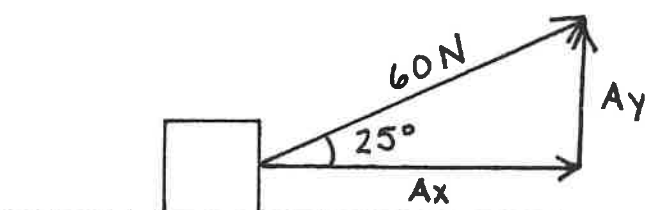
$A_x = 7.7\frac{m}{s}$

$A_y = A\sin\theta$

$A_y = (12\frac{m}{s})\sin 50°$

$A_y = 9.2\frac{m}{s}$

EXAMPLE 2:
A block is pulled across the floor by a 60 N force applied to a rope making an angle of 25° with the horizontal. Calculate the vertical and horizontal components of the force.



$A_y = A\sin\theta$

$A_y = (60N)\sin 25°$

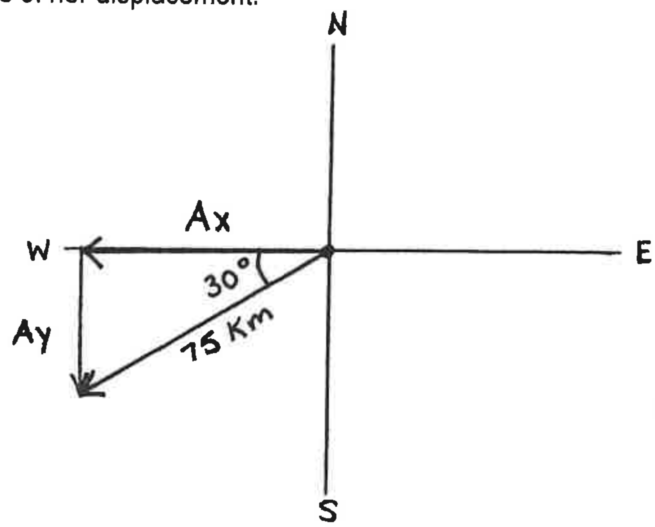
$A_y = 25.4N$

$A_x = A\cos\theta$

$A_x = (60N)\cos 60°$

$A_x = 54.4N$

EXAMPLE 3:
From her starting point, a student drives 30° south of west for 75 Km. Calculate the southern and western components of her displacement.



southern:

$A_y = A \sin \theta$

$A_y = (75 Km) \sin 30°$

$A_y = 37.5 Km$

eastern:

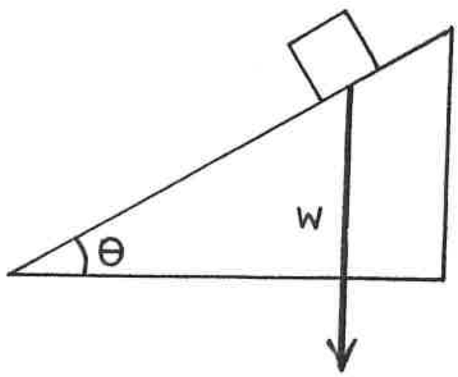
$A_x =$

$A_x = (75 Km) \cos 30°$

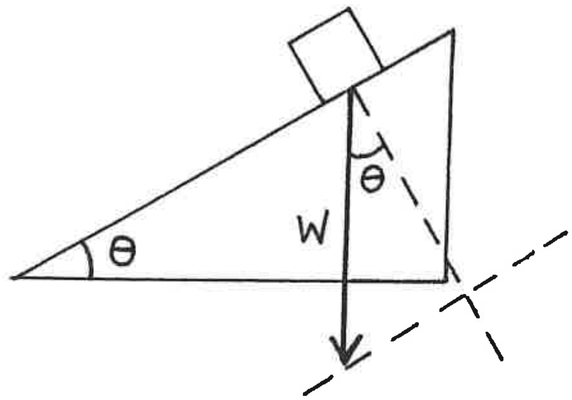
$A_x = 65 Km$

THE INCLINE PROBLEM

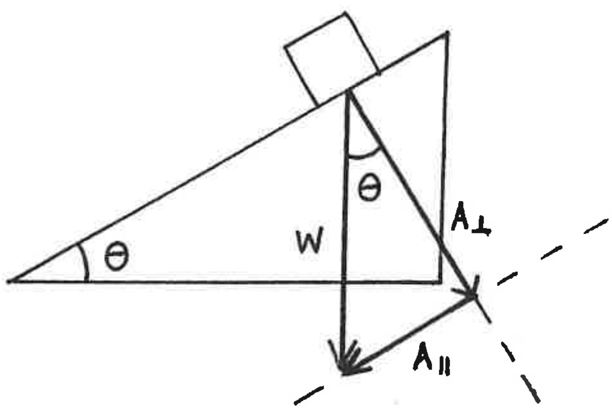
Incline problems are those in which an object is at rest or moves along an inclined (sloped) surface. They are just another example of a components problem, but they are a bit harder because the perpendicular components are not directed horizontally and vertically. Since the surface is at an angle, the perpendicular components will also be at angles. Some students find the vectors diagrams for incline problems more difficult to draw or the problems themselves more difficult to visualize, but these problems are really no different than any other components problem and obey the same algebraic rules in determining the components. In an incline problem, the resultant vector/hypotenuse is always the **weight** of the object on the incline. Weight is just another type of force; specifically it is the force of attraction of the Earth (or any planet) on an object. All gravitational forces (which will be discussed in a later unit) are attractive, so the planet always pulls the object toward the center of the planet. In other words, the weight vector will always be directed **down**. Whether an object is on a flat surface or an inclined surface, gravity pulls things **down**. It is always easiest to start your vector diagrams for incline problems by drawing the weight vector first.



After the weight vector is drawn in and labeled, the components can be drawn in. Since the components will not be horizontal and vertical, it is incorrect to use the notation $\mathbf{A_x}$ and $\mathbf{A_y}$ for the components of the weight. In drawing the two components of the weight vector, one will be parallel to the inclined surface and one will be perpendicular. Therefore, the correct notation for the parallel component will be **A** and the correct notation for the perpendicular component will be **A** . After the weight vector is drawn in, to draw the perpendicular component, take a ruler and starting from the tail of the weight vector, sketch in a line that is perpendicular to the incline. (The angle at the top is equal to θ due to similar triangles.)



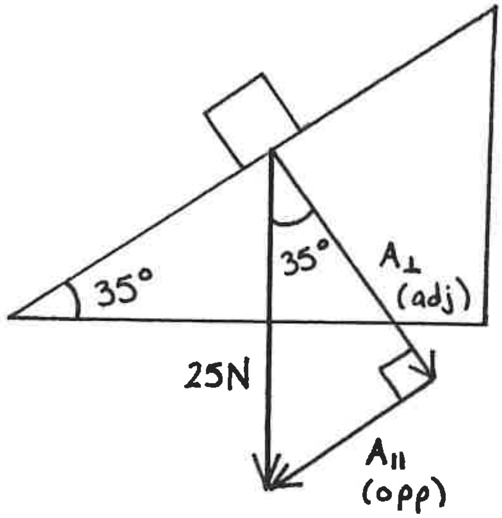
To draw the parallel component, move the ruler and align it such that it is parallel to the incline and touches the tip of the weight vector. What you have will still be a right triangle, but looks a little off because it is tilted.



Where the two dashed lines meet is where the perpendicular components ends; draw an arrowhead pointing down (away from the surface) at that point. This point is also where the tail of the parallel component is located; the arrowhead of the parallel component should be drawn pointed down the incline. Therefore, all three vectors are directed down: the weight vector is straight down (vertical line), the perpendicular component is down away from the surface and the parallel component is down parallel to the incline.

EXAMPLE 4:

A crate weighing 25 Newtons is held at rest on a 35° incline, as shown below. Calculate the components of the crate's weight that are parallel and perpendicular to the incline.



$$A_\parallel = A\sin\theta$$

$$A_\parallel = (25N)\sin 35°$$

$$A_\parallel = 14.3N$$

$$A_\perp = A\cos\theta$$

$$A_\perp = (25N)\cos 35°$$

$$A_\perp = 20.5N$$

TRIG REVIEW

Since a vector can be located in any quadrant, the signs of the components make it possible to determine in which quadrant a vector is located without necessarily having to draw out a vector diagram. A vector located in the first quadrant will have both positive x and y components (+x, +y). A vector located in the second quadrant will have a negative x component and a positive y component (-x, +y). A vector in the third quadrant will have both negative x and y components (-x, -y). A vector in the fourth quadrant will have a positive x component and a negative y component (+x, -y). From the components, one can calculate the magnitude of the resultant vector and the angle the vector makes, relative to the x axis. Since the components will form a right angle, the magnitude can be determined using the pythagorean theorem:

$$\mathbf{a^2 + b^2 = c^2}$$

The angle can be calculated using the trig function:

$$\mathbf{\tan \theta = y/x}$$

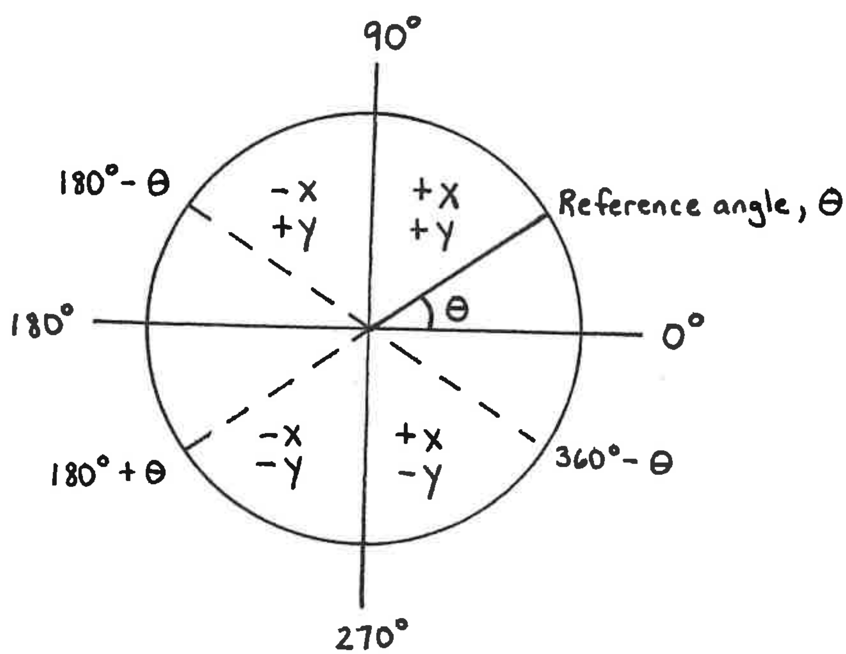
Once the angle is found, it can then be expressed using geographic directions or as an angle measured counterclockwise from the positive x axis. When using the tangent function, the angle given by the calculator will either be positive or negative depending on the signs of the components. From the value given, one can determine the **reference angle,** which is a first quadrant angle. The reference angle is simply the absolute value of the angle given by the calculator (it will always be positive). After finding the reference angle, the actual counterclockwise (cc) angle can be determined according to the following:

1st quadrant: the counterclockwise angle (θ) equals the reference angle

2nd quadrant: θ is equal to 180° - the reference angle

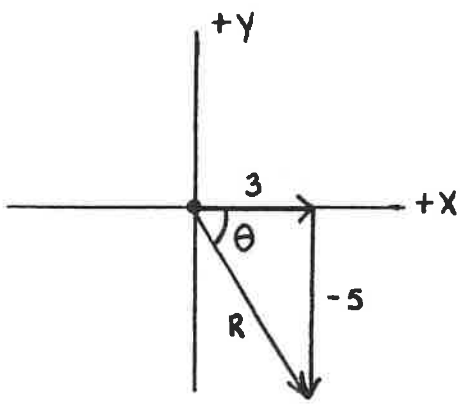
3rd quadrant: θ is equal to 180° + the reference angle

4th quadrant: θ is equal to 360° - the reference angle



EXAMPLE 1:
For each of the following pairs of components, determine the magnitude and direction of the resultant vector.

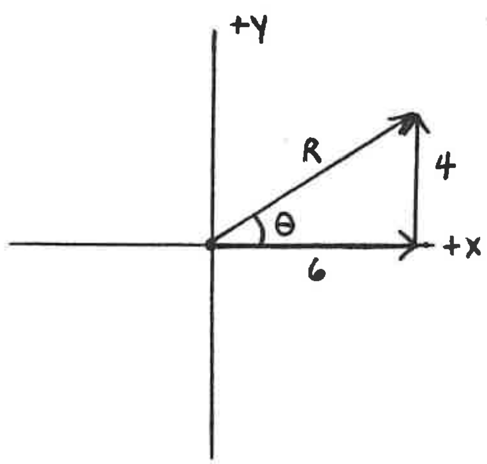
A. $A_x = 3$, $A_y = -5$



$a^2 + b^2 = c^2$
$(3)^2 + (-5)^2 = c^2$
$9 + 25 = c^2$
$c^2 = 34$
$c = 5.8$

$\tan\theta = \frac{y}{x}$
$\tan\theta = \frac{-5}{3} = -1.6$
$\theta = -59°$
ref angle $= 59°$
$(+x,-y) =$ 4th Quad
$\theta = 360° - 59° =$
301°cc

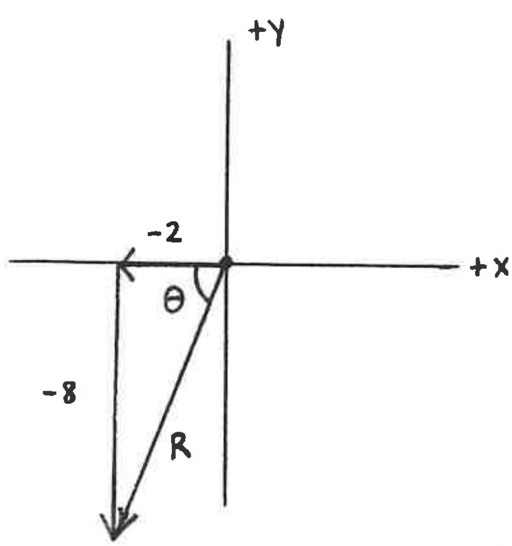
B. $A_x = 6$, $A_y = 4$



$a^2 + b^2 = c^2$
$(6)^2 + (4)^2 = c^2$
$36 + 16 = c^2$
$c^2 = 52$
$c = 7.2$

$\tan\theta = \frac{y}{x}$
$\tan\theta = \frac{4}{6} = 0.67$
$\theta = 34°$ (ref angle)
$(+x,+y) =$ 1st Quad
$\theta = 34°$cc

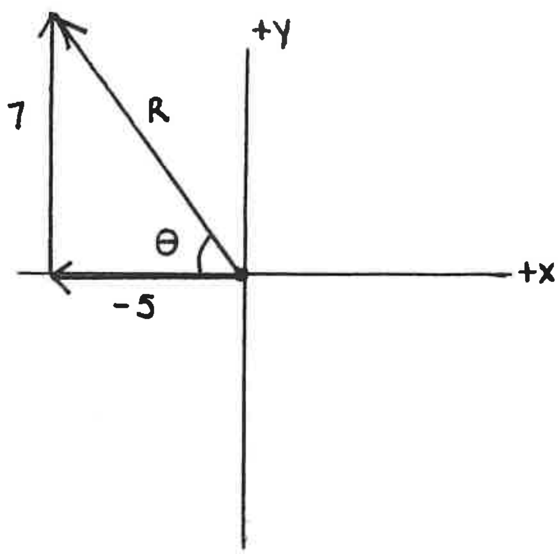
C. $A_x = -2$, $A_y = -8$



$a^2 + b^2 = c^2$
$(-2)^2 + (-8)^2 = c^2$
$4 + 64 = c^2$
$c^2 = 68$
$c = 8.2$

$\tan\theta = \frac{y}{x}$
$\tan\theta = \frac{-8}{-2} = +4$
$\theta = 76°$ (ref angle)
$(-x,-y) =$ 3rd Quad
$\theta = 180° + 76° =$
256°cc

D. $A_x = -5$, $A_y = 7$



$a^2 + b^2 = c^2$

$(-5)^2 + (7)^2 = c^2$

$25 + 49 = c^2$

$c^2 = 74$

$c = 8.6$

$\tan\theta = \frac{y}{x}$

$\tan\theta = \frac{7}{-5} = -1.4$

$\theta = -54°$

ref angle $= 54°$

$(-x, +y) =$ 2nd Quad

$\theta = 180° - 54° =$

$126°$ cc

It will be important to know which quadrant a vector is in, and to express the angles as counterclockwise from the positive x axis, when doing complex vector problems. These skills will make difficult problems progress much more quickly because drawing a vector diagram will not be necessary to do the problem. Interpreting positive and negative signs correctly can take the place of drawing all the vectors involved in each problem. However, you should always be ready to draw and label a complete vector diagram because they are frequently requested as part of the solution to a problem.

THE COMPONENTS METHOD

The components method is a means by which one can determine the magnitude and direction of a resultant vector or unknown vector in a problem. The components method can be used for <u>any</u> vector problem, but since it is a time consuming process, it is suggested that the best time to use it is when a problem contains more than two vectors and/or when the angles involved are not 0°, 90° or 180°. The components method involves setting up a chart (not mandatory, but it makes organizing all the numbers much easier!) with the given vectors and <u>counterclockwise</u> angles in the first two columns. Next, from these values determine all the x components using $\mathbf{A_x = A \cos\theta}$ and the y components using $\mathbf{A_y = A \sin\theta}$ and list these in the next two columns. Add up all the x components to get a value $\mathbf{R_x}$ and add up all the y components to get a value $\mathbf{R_y}$.

$$R_x = A_x + B_x + C_x + \ldots$$

$$R_y = A_y + B_y + C_y + \ldots$$

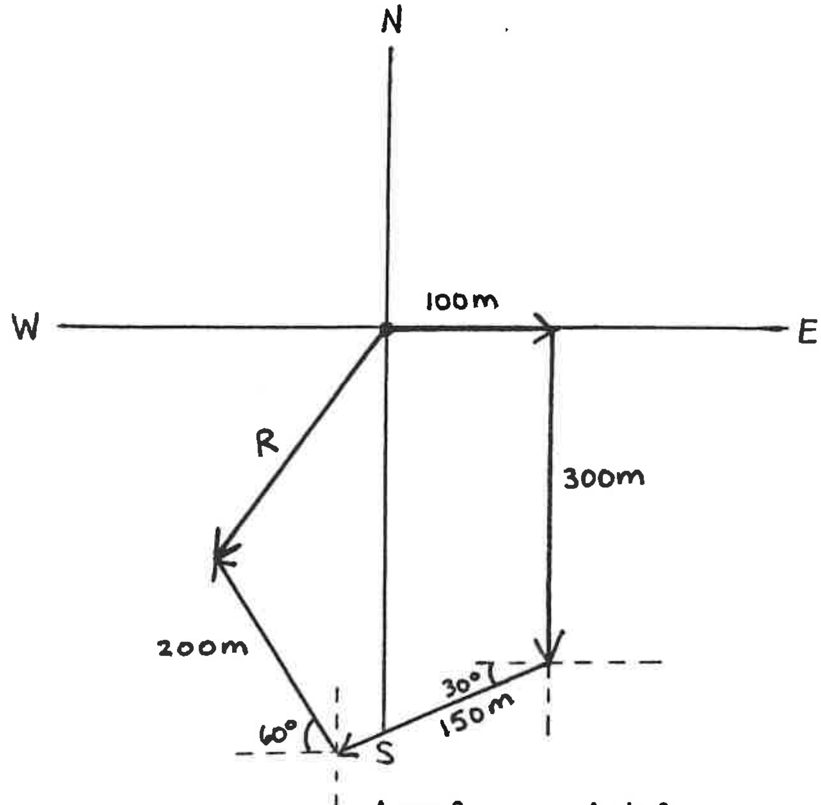
Using the values R_x and R_y (which essentially are the right legs of the resultant vector or unknown vector), the magnitude and direction of the resultant (or unknown vector) can be determined using the **pythagrorean theorem** and $\mathbf{\tan\theta = R_y/R_x}$. The signs of R_x and R_y will determine in which quadrant the resultant (or unknown vector) lies.

COMPONENTS METHOD CHART:

θ is measured counterclockwise
from the positive x axis!

Vector	θ	A cos θ X	A sin θ Y
A		A_x	A_y
B		B_x ADD	B_y ADD
C		C_x	C_y
D		D_x	D_y
		R_x	R_y

EXAMPLE 1:
A person hikes 100 meters east, then 300 meters south, then 30° south of west for 150 meters and then 60° north of west for 200 meters. Find the magnitude and direction of the person's resultant displacement.



Vector	θ	A cos θ X	A sin θ Y
100 m	0°	100 m	0 m
300 m	270°	0 m	-300 m
150 m	(180 + 30) 210°	-129.9 m	-75 m
200 m	(180 - 60) 120°	-100 m	173.2 m
		R_x = -129.9 m	R_y = -201.8 m

$a^2 + b^2 = c^2$

$(-129.9\,m)^2 + (-201.8\,m)^2 = c^2$

$c^2 = 57597.25\,m^2$

$c = 240\,m$

$\tan\theta = \frac{y}{x}$

$\tan\theta = \frac{-201.8\,m}{-129.9\,m} = 1.554$

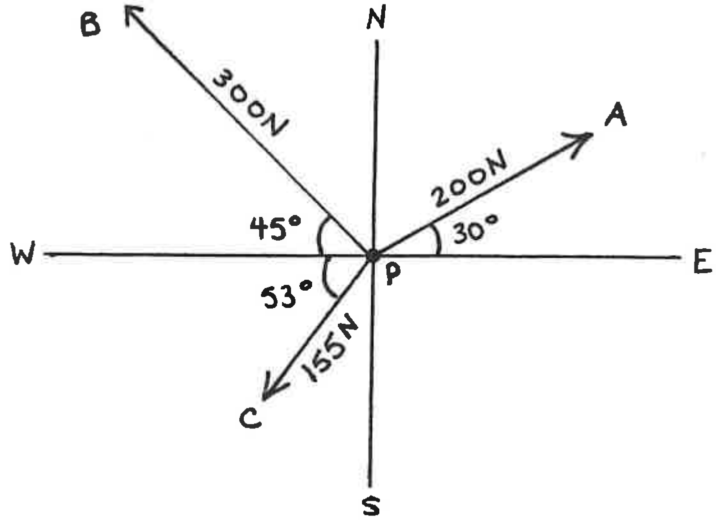
$\theta = 57°$

$(-x, -y)$ = 3rd Quad

$\theta = 180° + 57° = 237°$cc
(or 57° S of W)

EXAMPLE 2:

Three vectors, A, B and C act concurrently on point P, located at the origin, as shown below. Determine the magnitude and direction of the resultant force and the magnitude and direction of the equilibrant force, which will produce equilibrium.



Vector	θ	A cos θ X	A sin θ Y
200 N	30°	173.2 N	100 N
300 N	(180 – 45) 135°	-212.1 N	212.1 N
155 N	(180 + 53) 233°	-93.3 N	-123 N
		R_x = -132.2 N	**R_y = 188.3 N**

$a^2 + b^2 = c^2$

$(-132.2N)^2 + (188.3N)^2 = c^2$

$c^2 = 52938.04N^2$

$c = 230.1$ N

$\tan\theta = \frac{y}{x}$

$\tan\theta = \frac{188.3N}{-132.2N} = -1.424$

$\theta = -55°$

ref angle = 55°

$(-x, +y)$ = 2nd Quad

$\theta = 180° - 55° = 125°$ cc

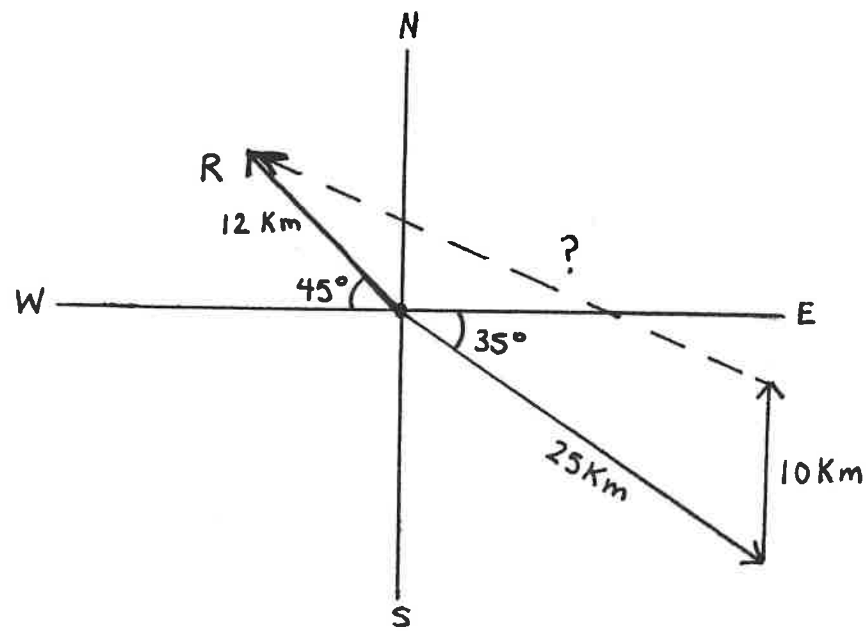
(or 55° N of W)

Resultant = 230.1N, 55° N of W
(or 125° cc)

Equilibrant = 230N, 55° S of E
(or 305° cc)

EXAMPLE 3:

An airplane flies 25 Km 35° south of east for the first part of a trip, continues 10 Km due north for the second part of the trip, and continues an unknown distance for the last part of the trip. If the plane lands 12 Km 45° north of west of its starting point, what is the magnitude and direction of the plane's third part of the trip?



Vector	θ	A cos θ X	A sin θ Y
25 Km	(360 – 35) 325°	+20.5 Km	-14.3 Km
10 Km	90°	0	+10 Km
?	?	X	Y
R = 12 Km	(180 – 45) 135°	R_x = -8.5 Km	R_y = +8.5 Km

To find X:

20.5 Km + 0 Km + X = -8.5Km

X = -29 Km

To find y:

-14.3 Km + 10 Km + y = +8.5 Km

y = +12.8 Km

$a^2 + b^2 = c^2$

$(-29\text{Km})^2 + (12.8\text{ Km})^2 = c^2$

C = 31.7Km

$\tan \theta = \frac{y}{x}$

$\tan \theta = \frac{12.8\text{Km}}{-29\text{ Km}} = -0.4414$

θ = -24°

ref angle = 24°

(-x, +y) = 2nd Quad

θ = 180° - 24° = 156° cc

(or 24° N of W)

EXAMPLE 4:
Three forces, **A, B** and **C** have the following magnitudes and directions:

A = 18 N, 42° west of south
B = 31 N, 61° west of north
C = 26 N, 12° south of east

(a) Determine the magnitude and direction of **A + B – C**

Vector	θ	A cos θ X	A sin θ Y
18 N	(270 – 42) 228°	-12.0 N	-13.4 N
31 N	(90 + 61) 151°	-27.1 N	+15.0 N
26 N	(180 – 12) 168°	-25.4 N	+5.4 N
		R_x = -64.5 N	R_y = +7.0 N

$a^2 + b^2 = c^2$

$(-64.5N)^2 + (7.0N)^2 = c^2$

$4160.25N^2 + 49N^2 = c^2$

$c^2 = 4209.25N^2$

$c = 64.9N$

$\tan\theta = \frac{Y}{X}$

$\tan\theta = \frac{7.0N}{-64.5N} = -0.1085$

$\theta = -6°$ ref angle = 6°

$(-x, +y)$ = 2nd Quad

$\theta = 180° - 6° = 174°$ cc

(or 6° N of W)

(b) Determine the magnitude and direction of **–A – B + C.**

Vector	θ	A cos θ X	A sin θ Y
18 N	(90 – 42) 48°	+12.0 N	+13.4 N
31 N	(270 + 61) 331°	+27.1 N	–15.0 N
26 N	(360 – 12) 348°	+25.4 N	-5.4 N
		R_x = +64.5 N	**R_y = – 7.0 N**

$a^2 + b^2 = c^2$

$(64.5N)^2 + (-7N)^2 = c^2$

$C = 64.9N$

$\tan\theta = \frac{y}{x}$

$\tan\theta = \frac{-7N}{64.5N} = -0.1085$

$\theta = -6°$ ref angle $= 6°$

$(+x, -y) = 4^{th}$ Quad

$\theta = 360° - 6° = 354°$cc

(or 6° S of E)

UNIT CONVERSIONS

An important skill in all of the sciences is unit conversions; the ability to convert one unit to another. There are some conversion factors that you would be expected to know, such as:

1 min = 60 sec	100 cm = 1 meter (m)
1 hour = 60 min	1000 mm = 1 meter (m)
1 day = 24 hours	1000 m = 1 Km
1 foot = 12 inches	

Other less common conversion factors would be provided to you, such as:

1 mile = 1.609 Km	1 mile = 5280 feet
1 Km = 0.6214 mile (mi)	1 Newton (N) = 0.2248 lbs
1 meter (m) = 3.281 ft	1 lb = 4.448 Newton (N)
1 inch = 2.54 cm	1 kg = 0.0685 slug

The key to unit conversions is to multiply the given number and unit by conversion factors so that the units in the numerator and denominator cancel out. This process is called the **FACTOR LABEL METHOD.** Square and cubic conversion factors can be developed from basic linear conversions (see examples 7 and 8).

Use the factor label method to convert the following quantities. Be sure to SHOW ALL WORK, including how the units cancel out!!

1) 45 Kg = 3.08 slugs

$$45\ \text{Kg} \times \frac{0.0685\ \text{slugs}}{1\ \text{Kg}} = 3.08\ \text{slugs}$$

2) 20 miles = 3.22×10^6 cm

$$20\ \text{miles} \times \frac{5280\ \text{ft}}{1\ \text{mile}} \times \frac{12\ \text{in}}{1\ \text{ft}} \times \frac{2.54\ \text{cm}}{1\ \text{in}} = 3.22 \times 10^6\ \text{cm}$$

3) 55 miles/hr = 24.6 meter/second

$$55\,\frac{mi}{hr} \times \frac{1\,hr}{60\,min} \times \frac{1\,min}{60\,sec} \times \frac{1.609\,Km}{1\,mi} \times \frac{1000\,m}{1\,Km} = 24.6\,\frac{m}{s}$$

4) 9.8 m/s^2 = 6.09×10^{-3} mi/s^2

$$9.8\,\frac{m}{s^2} \times \frac{3.281\,ft}{1\,m} \times \frac{1\,mile}{5280\,ft} = 6.09 \times 10^{-3}\,\frac{mi}{s^2}$$

5) 3×10^6 in/min = 4571.0 Km/hr

$$3 \times 10^6\,\frac{in}{min} \times \frac{60\,min}{1\,hr} \times \frac{1\,ft}{12\,in} \times \frac{1\,mi}{5280\,ft} \times \frac{1.609\,Km}{1\,mi} = 4571.0\,\frac{Km}{hr}$$

6) 5.3×10^{-3} in/sec = 3.01×10^{-4} mi/hr

$$5.3 \times 10^{-3}\,\frac{in}{sec} \times \frac{1\,ft}{12\,in} \times \frac{1\,mi}{5280\,ft} \times \frac{60\,sec}{1\,min} \times \frac{60\,min}{1\,hr} = 3.01 \times 10^{-4}\,\frac{mi}{hr}$$

7) 10 lbs/in² = 6.4×10^3 N/ft²

$$10\ \frac{\text{lbs}}{\text{in}^2} \times \frac{4.448\ \text{N}}{1\ \text{lb}} \times \frac{144\ \text{in}^2}{1\ \text{ft}^2} = 6.4 \times 10^3\ \frac{\text{N}}{\text{ft}^2}$$

12 in = 1 ft

$(12\ \text{in})^2 = (1\ \text{ft})^2$

$144\ \text{in}^2 = 1\ \text{ft}^2$

8) 2.11 L = 128.7 in³

(use 1 L = 1000 cm³)

$$2.11\ \text{L} \times \frac{1000\ \text{cm}^3}{1\ \text{L}} \times \frac{1\ \text{in}^3}{16.39\ \text{cm}^3} = 128.7\ \text{in}^3$$

1 in = 2.54 cm

$(1\ \text{in})^3 = (2.54\ \text{cm})^3$

$1\ \text{in}^3 = 16.39\ \text{cm}^3$